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# $\tilde{\pi}^0 \rightarrow \tilde{\gamma}\tilde{\gamma}$ in Dense QCD

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## Abstract

QCD superconductors in the color-flavor-locked (CFL) phase support light excitations (generalized pions) in the form of particle-particle or hole-hole excitations. We analyze the generalized process  $\tilde{\pi}^0 \rightarrow \tilde{\gamma}\tilde{\gamma}$  in the weak coupling limit and show that it is related to the recently suggested Wess-Zumino-Witten (WZW) term. In dense QCD, the radiative decay of the generalized pion is constrained by geometry and vanishes at large density.

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**1.** At high density, QCD exhibits a superconducting phase with novel and nonperturbative phenomena [1, 2]. For degenerate quark masses the QCD superconductor breaks color and flavor spontaneously with the occurrence of Goldstone modes. These modes were analyzed recently using effective Lagrangians [3, 4, 5] (zero size) and bound state equations [6, 7] (finite size).

In the color-flavor-locked (CFL) phase the Goldstone modes are characterized by a Wess-Zumino-Witten (WZW) term with a generalized axial-anomaly [3]. In the present letter we analyze the decay process  $\tilde{\pi}^0 \rightarrow \tilde{\gamma}\tilde{\gamma}$  of the generalized pion  $\tilde{\pi}^0$  into a pair of modified photons  $\tilde{\gamma}$  using weak-coupling arguments at large quark chemical potential. After recalling some features of the QCD superconductor, we proceed to an analysis of the triangle anomaly in the CFL phase. We derive an explicit result for the amplitude of  $\tilde{\pi}^0 \rightarrow \tilde{\gamma}\tilde{\gamma}$  in the limit of large quark density. We show that our result agrees with the recently suggested WZW term [3]. The decay amplitude and the color-flavor anomalies in the CFL phase are related at asymptotic densities.

**2.** In the QCD superconductor, the quarks are gapped. Their propagation in the chiral limit is given in the Nambu-Gorkov formalism by [6, 8, 9]

$$\begin{aligned}
S_{11}(q) &= \frac{1}{i} \langle \psi(q) \bar{\psi}(q) \rangle = \left[ \Lambda^+(\mathbf{q}) \gamma^0 \frac{q_0 + q_{||}}{q_0^2 - \epsilon_q^2} \Lambda^-(\mathbf{q}) + \Lambda^-(\mathbf{q}) \gamma^0 \frac{q_0 - q_{||} - 2\mu}{q_0^2 - \bar{\epsilon}_q^2} \Lambda^+(\mathbf{q}) \right] \\
S_{12}(q) &= \frac{1}{i} \langle \psi(q) \bar{\psi}_C(q) \rangle = - \left[ \Lambda^+(\mathbf{q}) \mathbf{M}^\dagger \frac{G^*(q)}{q_0^2 - \epsilon_q^2} \Lambda^+(\mathbf{q}) + \Lambda^-(\mathbf{q}) \mathbf{M}^\dagger \frac{\bar{G}^*(q)}{q_0^2 - \bar{\epsilon}_q^2} \Lambda^-(\mathbf{q}) \right] \\
S_{21}(q) &= \frac{1}{i} \langle \psi_C(q) \bar{\psi}(q) \rangle = \left[ \Lambda^-(\mathbf{q}) \mathbf{M} \frac{G(q)}{q_0^2 - \epsilon_q^2} \Lambda^-(\mathbf{q}) + \Lambda^+(\mathbf{q}) \mathbf{M} \frac{\bar{G}(q)}{q_0^2 - \bar{\epsilon}_q^2} \Lambda^+(\mathbf{q}) \right] \\
S_{22}(q) &= \frac{1}{i} \langle \psi_C(q) \bar{\psi}_C(q) \rangle = \left[ \Lambda^-(\mathbf{q}) \gamma^0 \frac{q_0 - q_{||}}{q_0^2 - \epsilon_q^2} \Lambda^+(\mathbf{q}) + \Lambda^+(\mathbf{q}) \gamma^0 \frac{q_0 + q_{||} + 2\mu}{q_0^2 - \bar{\epsilon}_q^2} \Lambda^-(\mathbf{q}) \right]
\end{aligned}$$

with the momentum  $q_{||} = (|\mathbf{q}| - \mu)$  measured parallel and relative to the Fermi momentum  $|\mathbf{p}_F| = \mu$ , the squared particle/hole energies  $\epsilon_q^2 = q_{||}^2 + \mathbf{M}^\dagger \mathbf{M} |G(q)|^2$  and the squared anti-particle/anti-hole energies  $\bar{\epsilon}_q^2 = (q_{||} + 2\mu)^2 + \mathbf{M}^\dagger \mathbf{M} |\bar{G}(q)|^2$ . The operators  $\Lambda^\pm(\mathbf{q}) = \frac{1}{2}(1 \pm \boldsymbol{\alpha} \cdot \hat{\mathbf{q}})$  are the positive and negative energy projectors. In the CFL phase,  $\mathbf{M} = \epsilon_f^a \epsilon_c^a \gamma_5 = \mathbf{M}^\dagger$  is the locking matrix in the color-flavor (*c-f*) sector with  $(\epsilon^a)^{bc} = \epsilon^{abc}$  the totally antisymmetric tensor. To leading logarithm accuracy, the gap function  $G(q)$  is solely a real-valued function of  $q_{||}$  given by [9, 10, 11]

$$G(x) = G_0 \sin\left(\frac{\pi x}{2x_0}\right) = G_0 \sin\left(h_* x / \sqrt{3}\right). \quad (1)$$

Here the logarithmic scales are defined as  $x = \ln(\Lambda_*/q_{||})$  and  $x_0 = \ln(\Lambda_*/G_0)$ , with  $\Lambda_* = (4\Lambda_\perp^6/\pi m_E^5)$  in terms of the transversal cutoff  $\Lambda_\perp = 2\mu$  and the electric screening mass

$m_E = \sqrt{\frac{N_F}{2\pi^2}} g\mu$ , where  $g$  is the strong coupling constant and  $N_F$  the number of flavors. Also, we have defined  $h_* x_0/\sqrt{3} = \pi/2$ , where  $h_* = g/(\sqrt{6}\pi)$ , and  $G_0$  as

$$G_0 \approx \left( \frac{4\Lambda_{\perp}^6}{\pi m_E^5} \right) e^{-\frac{\sqrt{3}\pi}{2h_*}}. \quad (2)$$

The wave-functions of the pseudoscalar excitations in the CFL phase have been discussed in [6, 7]. In particular, in leading logarithm approximation, the pseudoscalar vertex operator of ‘isospin’  $A$  for a pair of particles or holes with momenta  $P/2 \pm p$  is given by

$$\Gamma_{\pi}^A(p, P) = \frac{1}{F_T} \begin{pmatrix} 0 & -i\gamma_5 \Gamma_{PS}^*(p, P) (\mathbf{M}^A)^{\dagger} \\ i\gamma_5 \Gamma_{PS}(p, P) \mathbf{M}^A & 0 \end{pmatrix} \quad (3)$$

with  $\mathbf{M}^A = \mathbf{M}^{a\alpha} (\tau^A)^{a\alpha}$  and  $\mathbf{M}^{a\alpha} = \epsilon_f^a \epsilon_c^{\alpha} \gamma_5$ . Note that  $(\mathbf{M}^A)^{\dagger} = \epsilon_f^a \epsilon_c^{\alpha} \gamma_5 (\tau^{A*})^{a\alpha} = \epsilon_f^a \epsilon_c^{\alpha} \gamma_5 (\tau^A)^{a\alpha}$ . In the chiral limit, the pseudoscalar vertex reduces to the gap, i.e.  $\Gamma(p, 0) = G(p)$ . Throughout,  $F_T = \mu/\pi$  refers to the temporal pion decay constant [3, 5, 6]. The CFL phase supports other collective excitations as well [7]. They are not needed for the rest of our discussion.

**3.** In the CFL phase the ordinary photon is screened, and the gluons are either screened or higgsed. However, it was pointed out in [12] that the CFL phase is transparent to a modified or tilde photon,

$$\tilde{A}_{\mu} = A_{\mu} \cos\theta + H_{\mu} \sin\theta, \quad (4)$$

where  $\cos\theta = g/\sqrt{e^2 + g^2}$ ,  $A_{\mu}$  is the photon field coupling to the charge matrix  $e \mathbf{Q}_{em}$  of the quarks and  $H_{\mu}$  is the gluon field for  $U(1)_Y$  where  $g \mathbf{Y}$  is the color-hypercharge matrix <sup>#1</sup> and  $\sin\theta = -e/\sqrt{e^2 + g^2}$ .  $\tilde{A}_{\mu}$  carries color-flavor and tags to the charges of the Goldstone modes. The quark coupling to the tilde photon is in units of  $\tilde{e} = e \cos\theta$ . As a result, the CFL phase is characterized by generalized flavor-color anomalies [3]. In this section, we show how the triangle ‘anomaly’ emerges from a direct calculation of the  $\tilde{\pi}^0 \rightarrow \tilde{\gamma}\tilde{\gamma}$  in the leading logarithm approximation.

The contribution of (4) to the triangle graph is

$$\begin{aligned} \mathcal{T}_{\mu\nu}^A(K_1, K_2) = & - \int \frac{d^4 q}{(2\pi)^4} \text{Tr} \left( i\Gamma_{\pi}^A(q, P) i\mathbf{S}(q - \frac{P}{2}) i\tilde{\mathbf{Q}}_{\mu} i\mathbf{S}(q + \frac{Q}{2}) i\tilde{\mathbf{Q}}_{\nu} i\mathbf{S}(q + \frac{P}{2}) \right) \\ & + [(K_1 \leftrightarrow K_2) \text{ and } (\mu \leftrightarrow \nu)] \end{aligned} \quad (5)$$

with  $P = K_1 + K_2$  and  $Q = K_2 - K_1$ . The spin-color-flavor vertex due to (4) reads

$$\tilde{\mathbf{Q}}_{\mu} = \tilde{e} \boldsymbol{\rho}_3 \gamma_{\mu} \text{diag}(\tilde{\mathbf{Q}}, \tilde{\mathbf{Q}}^T) = \tilde{e} \gamma_{\mu} \boldsymbol{\rho}_0 \tilde{\mathbf{Q}} = \tilde{e} \gamma_{\mu} \boldsymbol{\rho}_0 (\mathbf{Q}_{em} \otimes \mathbf{1}_c - \mathbf{1}_f \otimes \mathbf{Y}). \quad (6)$$

<sup>#1</sup>Without loss of generality, the representation of  $\mathbf{Y}$  can be chosen to be identical to the one of  $\mathbf{Q}_{em}$ .

Here  $\rho_3$  and  $\rho_0$  are Pauli matrices acting on the Nambu-Gorkov indices. We assume that the indices  $\mu$  and  $\nu$  are eventually contracted with the polarizations  $\mathcal{E}_T^\mu$  and  $\mathcal{E}_T^\nu$  (space-like). The result for (5) after inserting (1) and (3) and lengthy algebra is

$$\begin{aligned} \mathcal{T}_{\mu\nu}^A(K_1, K_2) &= 2i \frac{\tilde{e}^2}{F_T} \int \frac{d^4 q}{(2\pi)^4} \Gamma_{PS}(p, P) \left[ \Sigma_{\mu\nu}^{A(0)}(P_i, \mathbf{M}) + \Sigma_{\mu\nu}^{A(1)}(P_i, \mathbf{M}) + \mathcal{O}(1/\mu^3) \right] \\ &\quad + \left( P_{i+} \leftrightarrow P_{i-} \text{ or } P_{i-} - 2\mu \leftrightarrow P_{i+} + 2\mu \text{ and } \Lambda^+ \leftrightarrow \Lambda^-, \mathbf{M}^{(A)} \leftrightarrow \mathbf{M}^{(A)\dagger} \right) \end{aligned} \quad (7)$$

with  $P_1 \equiv q - P/2$ ,  $P_2 \equiv q + Q/2$ ,  $P_3 \equiv q + P/2$ ,  $P_{i\pm} \equiv P_{i0} \pm P_{i||}$ . The explicit form of  $\Sigma_{\mu\nu}^{A(0)}$  is

$$\begin{aligned} \Sigma_{\mu\nu}^{A(0)} &= \left\{ \text{Tr}_{cf}[\mathbf{M}^A \tilde{\mathbf{Q}} \tilde{\mathbf{Q}} \mathbf{M}^\dagger] P_{1+} P_{2+} G(P_3) \right. \\ &\quad - \text{Tr}_{cf}[\mathbf{M}^A \tilde{\mathbf{Q}} \mathbf{M}^\dagger \tilde{\mathbf{Q}}] P_{1+} G(P_2) P_{3-} \\ &\quad + \text{Tr}_{cf}[\mathbf{M}^A \mathbf{M}^\dagger \tilde{\mathbf{Q}} \tilde{\mathbf{Q}}] G(P_1) P_{2-} P_{3-} \\ &\quad \left. - \text{Tr}_{cf}[\mathbf{M}^A \mathbf{M}^\dagger \tilde{\mathbf{Q}} \mathbf{M} \tilde{\mathbf{Q}} \mathbf{M}^\dagger] G(P_1) G(P_2) G(P_3) \right\} \\ &\quad \times \frac{1}{\Delta(P_1) \Delta(P_2) \Delta(P_3)} \text{Tr}[\gamma_5 \Lambda^+(\mathbf{P}_1) \gamma_\mu \Lambda^-(\mathbf{P}_2) \gamma_\nu \Lambda^+(\mathbf{P}_3)]. \end{aligned} \quad (8)$$

The explicit form of  $\Sigma_{\mu\nu}^{A(1)}$  is

$$\begin{aligned} \Sigma_{\mu\nu}^{A(1)} &= \left\{ \frac{1}{\Delta(P_1) \bar{\Delta}(P_2) \Delta(P_3)} \text{Tr}_{cf}[\mathbf{M}^A \tilde{\mathbf{Q}} \tilde{\mathbf{Q}} \mathbf{M}^\dagger] P_{1+} (P_{2-} - 2\mu) G(P_3) \right. \\ &\quad - \frac{1}{\bar{\Delta}(P_1) \Delta(P_2) \bar{\Delta}(P_3)} \text{Tr}_{cf}[\mathbf{M}^A \tilde{\mathbf{Q}} \mathbf{M}^\dagger \tilde{\mathbf{Q}}] (P_{1-} - 2\mu) G(P_2) (P_{3+} + 2\mu) \\ &\quad - \frac{1}{\Delta(P_1) \bar{\Delta}(P_2) \Delta(P_3)} \text{Tr}_{cf}[\mathbf{M}^A \tilde{\mathbf{Q}} \mathbf{M}^\dagger \tilde{\mathbf{Q}}] P_{1+} \bar{G}(P_2) P_{3-} \\ &\quad + \frac{1}{\Delta(P_1) \bar{\Delta}(P_2) \Delta(P_3)} \text{Tr}_{cf}[\mathbf{M}^A \mathbf{M}^\dagger \tilde{\mathbf{Q}} \tilde{\mathbf{Q}}] G(P_1) (P_{2+} + 2\mu) P_{3-} \\ &\quad \left. - \frac{1}{\Delta(P_1) \bar{\Delta}(P_2) \Delta(P_3)} \text{Tr}_{cf}[\mathbf{M}^A \mathbf{M}^\dagger \tilde{\mathbf{Q}} \mathbf{M} \tilde{\mathbf{Q}} \mathbf{M}^\dagger] G(P_1) \bar{G}(P_2) G(P_3) \right\} \\ &\quad \times \text{Tr}[\gamma_5 \Lambda^+(\mathbf{P}_1) \gamma_\mu \Lambda^+(\mathbf{P}_2) \gamma_\nu \Lambda^+(\mathbf{P}_3)]. \end{aligned} \quad (9)$$

We have defined  $\Delta(P) = P^2 - \epsilon_P^2$ ,  $\bar{\Delta}(P) = P^2 - \bar{\epsilon}_P^2$ , and made use of the fact that in the generalized pion CM frame  $\mathbf{P}_1 = \mathbf{P}_3$ , thereby reducing the spin contributions <sup>#2</sup>.

The various contributions in  $\Sigma^{(0)}$  arise from particles and holes. Naively,  $\Sigma^{(0)}$  generates a term of order  $\mu^2$  as it receives contribution from the full Fermi surface. We show below that this contribution is identically zero after integration. The various contributions to  $\Sigma^{(1)}$  involve at least an antiparticle. Since  $\bar{\Delta} \approx -4\mu^2$ , the overall contribution is of order  $\mu^0$  after integrating over the Fermi surface <sup>#3</sup>. It can be shown that the non-vanishing

<sup>#2</sup>Because of  $\gamma_5 \Lambda^\pm(\mathbf{P}_1) = \gamma_5 \Lambda^\pm(\mathbf{P}_3) = \Lambda^\pm(\mathbf{P}_3) \gamma_5$ , the left-out spin contributions project to zero after a cyclic permutation under the Dirac trace.

<sup>#3</sup>The terms linear and quadratic in  $\mu$  cancel when (9) is inserted into (7).

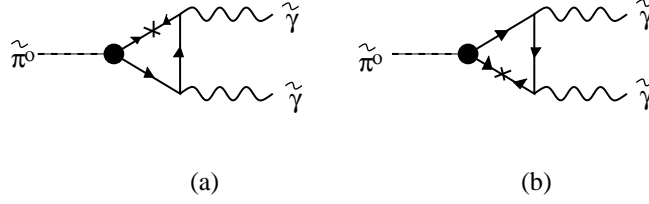


Figure 1: Leading contributions to  $\tilde{\pi}^0 \rightarrow \tilde{\gamma}\tilde{\gamma}$  in the CFL phase.

contributions arising from the various terms in  $\Sigma^{(1)}$  are those displayed in Fig. 1, where the cross refers to the pair-condensate insertion. They correspond to the first and fourth line of (9) (and first and third line of (8)) when inserted into (7), and they saturate the photo-decay of the pion in dense QCD. Finally, we note the different structure in the Lorentz projectors in (8) and (9). To the order considered, we use  $\mathbf{M}^\dagger \mathbf{M} = 1$  in  $\Delta$  and  $\bar{\Delta}$  <sup>#4</sup>. The overall factor of 2 in front of (7) comes from the triangle graph with crossed tilde-photon legs. The various color-flavor traces yield equal contributions modulo overall signs. Specifically,

$$\begin{aligned}
\text{Tr}_{cf} \left( \mathbf{M}^A \tilde{\mathbf{Q}} \tilde{\mathbf{Q}} \mathbf{M}^\dagger \right) &= -2N_f \text{Tr} \left( \tau^A \tilde{\mathbf{Q}}_*^2 \right) \\
\text{Tr}_{cf} \left( \mathbf{M}^A \tilde{\mathbf{Q}} \mathbf{M}^\dagger \tilde{\mathbf{Q}} \right) &= +2N_f \text{Tr} \left( \tau^A \tilde{\mathbf{Q}}_*^2 \right) \\
\text{Tr}_{cf} \left( \mathbf{M}^A \mathbf{M}^\dagger \tilde{\mathbf{Q}} \tilde{\mathbf{Q}} \right) &= -2N_f \text{Tr} \left( \tau^A \tilde{\mathbf{Q}}_*^2 \right) \\
\text{Tr}_{cf} \left( \mathbf{M}^A \mathbf{M}^\dagger \tilde{\mathbf{Q}} \mathbf{M} \tilde{\mathbf{Q}} \mathbf{M}^\dagger \right) &= +2N_f \text{Tr} \left( \tau^A \tilde{\mathbf{Q}}_*^2 \right) ,
\end{aligned} \tag{10}$$

where we have defined

$$\tilde{\mathbf{Q}}_* = \frac{1}{3} \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} . \tag{11}$$

The overall factor of 2 in (10) stems from an equal contribution due to either flavor or color, since in the CFL phase  $N_f = N_c$ .

Inserting (10) into (8) and performing the energy integration over the Fermi surface yield

$$\mathcal{T}_{\mu\nu}^{A(0)}(K_1, K_2) = \mathcal{R}_0 \mathcal{I}_0 \frac{\tilde{e}^2}{2\pi^2 F_T} \text{Tr} \left( \tau^A \tilde{\mathbf{Q}}_*^2 \right) \epsilon_{\mu\nu\alpha\beta} K_1^\alpha K_2^\beta , \tag{12}$$

where  $\mathcal{R}_0$  is an overall constant and  $\mathcal{I}_0$  is an integral of the form

$$\int d^4 q \frac{G^2(q)}{\Delta^3(q)} \sim \frac{\mu^2}{G_0^2} . \tag{13}$$

The  $\mu^2$  on the r.h.s. of (13) arises from integrating over the full Fermi surface. This contribution implies a decay rate that increases substantially in dense matter. Careful algebra, however, shows that after integration over the Fermi surface, the sum of the various

<sup>#4</sup>Under the assumption that  $U_A(1)$  is broken by the anomaly.

contributions to  $\Sigma^{(0)}$  yields  $\mathcal{R}_0 = 0$ . The decay rate is driven by  $\Sigma^{(1)}$  to leading order, as we now show.

To facilitate the calculation of the various contributions to  $\Sigma^{(1)}$ , we choose the CM kinematics with  $P_1 = (q^0 - M/2, \vec{q})$ ,  $P_2 = (q^0 + Q^0/2, \vec{q} + \vec{Q}/2)$ ,  $P_3 = (q^0 + M/2, \vec{q})$ . After some algebra we obtain

$$\mathcal{T}_{\mu\nu}^{A(1)}(K_1, K_2) = 2 \frac{\tilde{e}^2}{F_T} \frac{N_f}{3} \frac{\mathcal{I}_1}{\mu^2} \text{Tr} \left( \tau^A \tilde{\mathbf{Q}}_*^2 \right) \epsilon_{\mu\nu\alpha\beta} K_1^\alpha K_2^\beta. \quad (14)$$

Note that the  $1/3$  results from an angular integration over the 3-momenta through the Fermi surface, while

$$\mathcal{I}_1 = \int \frac{d^4 q}{(2\pi)^4} \frac{G^2(q)}{\Delta^2(q)} = \frac{i}{8} F_T^2 = \frac{i \mu^2}{8\pi^2} \quad (15)$$

is proportional to the square of the temporal pion decay constant [5, 6]. The presence of the antiparticle yields a  $\mu$ -independent contribution <sup>#5</sup>. Note that the various contributions involving the antiparticle gap  $\bar{G}$  in  $\Sigma^{(1)}$  contribute zero after integration. The same is true for the terms with any pair-condensate insertion on the fermion leg in between the two photon vertices. Hence,

$$\mathcal{T}_{\mu\nu}^A(K_1, K_2) = i \frac{\tilde{e}^2}{F_T} \frac{1}{4\pi^2} \text{Tr} \left( \tau^A \tilde{\mathbf{Q}}_*^2 \right) \epsilon_{\mu\nu\alpha\beta} K_1^\alpha K_2^\beta \quad (16)$$

to leading order in the density. We have checked that the possible vertex corrections to (3) are subleading in the decay rate. Because of the  $F_T$ -dependence, eq. (16) suggests that the radiative decay of the generalized pion vanishes as  $1/\mu$  in dense matter.

This result, based on our explicit calculation in the CFL phase to leading order, is in agreement with the result suggested by the generalized WZW term [3],

$$\partial^\mu \mathbf{A}_\mu^3 = -\frac{\tilde{e}^2}{96\pi^2} \epsilon_{\mu\nu\rho\sigma} \tilde{F}^{\mu\nu} \tilde{F}^{\rho\sigma} \quad (17)$$

with  $\tilde{F}^{\mu\nu}$  the field strength associated to (4) <sup>#6</sup>. The finite-size of the generalized pion does not upset the geometrical normalization of the anomaly in the generalized WZW form [3]. A direct comparison to the  $\mu = 0$  radiative decay of the usual pion in QCD, shows that (17) is off by a factor of  $1/3$ . As originally noted in [3], the color-flavor anomaly in the CFL phase is no longer multiple of  $N_c$  due to the color-flavor locking in the triangle graph, hence a factor of  $1/3$ . This point is particularly explicit in the various traces in (10).

**4.** In the CFL phase ordinary photons are screened, and perturbative gluons are either screened or higgsed. The anomalous decay of the generalized pions occurs via the

<sup>#5</sup>The same observation applies to most of the mixing processes discussed in [7] and found to vanish to leading order.

<sup>#6</sup>The normalization of the  $\pi^0 \gamma \gamma$  amplitude is 8 times bigger than the normalization of the corresponding effective Lagrangian or of the pertinent anomaly (17). Furthermore,  $\text{Tr} \left( \tau^3 \tilde{\mathbf{Q}}_*^2 \right) = 1/3$ .

tilde-photon. We have provided a direct calculation of the radiative decay  $\tilde{\pi}^0 \rightarrow \tilde{\gamma}\tilde{\gamma}$  in this phase using weak-coupling arguments. To leading logarithm accuracy our result is exact and in agreement with the normalization suggested by the generalized WZW term given in [3]. Much like in the vacuum, the radiative decay of the generalized pions is dictated by geometry in leading order, and vanishes at asymptotic densities. Clearly our analysis extends to other anomalous as well as nonanomalous processes, and should prove useful for the analysis of emission rates in dense QCD.

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